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M E M O R A N D U M

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CONSISTENCY TESTING OF THERMAL BENCHMARK LATTICE EXPERIMENTS

INTRODUCTION

Results of thermal benchmark lattice integral measurements are widely used to test analytical methods used in lattice analysis and assess the worth of differential neutron cross section data. A set of experimental measurements are designated a "benchmark" if they are clean, well documented and display internal consistency, at least qualitatively. A literature search has not revealed any general quantitative test that has been applied to experimental results to demonstrate consistency, although some experiments must have been subjected to some form or other of quantitative test.

An equation based on a two group neutron balance condition that is capable of revealing the quantitative consistency (or inconsistency) of reported thermal benchmark lattice integral parameters has been suggested.[1] This equation is used in conjunction with a second equation in the following discussion to assess the consistency (or inconsistency) of: 1) several Cross Section Evaluation Working Group (CSEWG) defined thermal benchmark lattices, 2) SRL experiments on the Mark 5R and Mark 15 lattices, and 3) a few D_2O lattices encountered as proposed thermal benchmark lattices.

SUMMARY

Nineteen thermal benchmark lattice experiments were subjected to a quantitative test of consistency between the reported experimental integral parameters. Results of this testing showed only two lattice experiments to be generally useful, three lattice experiments to be of limited usefulness, three lattice experiments to be potentially useful, and 11 lattice experiments to be not useful. These results are tabulated on the next page with the lattices identified.

Lattice Name	Type of Lattice	Consistency*	Usefulness[†]
CSEWG Benchmarks TRX-1 and TRX-2[4]	One Region H_2O Criticals	Good	Generally
CSEWG Benchmarks TRX-3 and TRX-4[4]	Two Region H_2O Criticals	Not determined	Limited
CSEWG Benchmarks MIT-1, MIT-2, MIT-3[4]	D_2O Sub-criticals	Poor	None
SRL Mark 5R Experiments[7] (4 Lattices)	D_2O Sub-criticals	Poor**	None
SRL Mark 15 Experiments[7] (4 Lattices)	D_2O Sub-criticals	Poor**	None
MIT 0.387" Rod Experiments[8] (3 Lattices)	D_2O Sub-criticals	Fair	Potentially
CRNL 20.0 cm Pitch ZEEP Lattice[9]	D_2O Critical	Good	Limited

* Consistency between activation measurements and material bucklings.

** Effort is underway through the use of critical experiments to identify the source of the inconsistency. If the source could be shown to be systematic, the inconsistency might be eliminated making this body of information useful.

[†] Usefulness for testing calculation methods and cross sections.

The sub-critical lattices (all D_2O moderated) in this study showed systematic inconsistencies in activation parameters and universally high material bucklings. No sub-critical lattice experiment in this study was found to be fully consistent, while all of the critical experiments for which a full set of measurements was made showed generally good consistency.

The most important numerical results of this study are shown in Tables IX and X.

The RAHABR[2] lattice physics module used in this study gave good agreement with experiment in the few cases for which the experimental results were internally consistent. Additional accurate experiments are required to validate RAHABR for lattices more typical of SRL operation and serve as a basis for small cross section adjustments. The current Mark 15 critical experiments were designed to meet the most urgent part of this need, but additional experiments may be desirable.

DISCUSSION

The discussion to follow will first derive an expression for the lattice material buckling in terms of the experimentally determined activation parameters plus other calculable constants. This derivation appears in Section I. Section II describes how the equations of Section I were applied to determine the consistency (or inconsistency) of a thermal lattice experiment. Section III presents the detailed results for each thermal lattice experiment in this study. Section IV summarizes the results shown in Section III into a more general set of conclusions. Section V, the last section, examines the sub-critical experiments in an attempt to determine any systematic features of the consistency testing results.

I. Two Group Expression for Material Buckling

For a uniform lattice with ^{235}U and ^{238}U the only fissionable materials present, the two group neutron balance equation that must hold at critical (or steady state sub-critical with constant source) is

$$(D_1\phi_1 + D_2\phi_2)B_m^2 + \Sigma_{a_1}\phi_1 + \Sigma_{a_2}\phi_2 = \nu\Sigma_{f_1}\phi_1 + \nu\Sigma_{f_2}\phi_2 \tag{1}$$

Dividing equation 1 by $\Sigma_{f_2}\phi_2$ and rewriting slightly yields

$$\left\{\frac{D_1\phi_1/\phi_2 + D_2}{\Sigma_{f_2}}\right\} B_m^2 = \frac{\left\{\nu\Sigma_{f_1}^{25}\phi_1 + \nu\Sigma_{f_1}^{28}\phi_1 + \nu\Sigma_{f_2}^{25}\phi_2 - \Sigma_{C_1}^{25}\phi_1 - \Sigma_{C_1}^{28}\phi_1 - \Sigma_{C_1}^{O}\phi_1 - \Sigma_{f_1}^{25}\phi_1 - \Sigma_{f_1}^{28}\phi_1\right\}}{\Sigma_{f_2}\phi_2}$$

$$-\frac{\Sigma_{C_2}^{25}\phi_2 + \Sigma_{C_2}^{28}\phi_2 + \Sigma_{C_2}^{O}\phi_2 + \Sigma_{f_2}^{25}\phi_2}{\Sigma_{f_2}\phi_2} \tag{1a}$$

where $\Sigma_{C_n}^{O}$ represents capture in all materials other than ^{235}U and ^{238}U.

Definitions of activation parameters are:

^{238}U(n,γ) capture ratio $\quad \rho^{28} = \dfrac{\text{Epithermal } ^{238}\text{U captures}^{\dagger}}{\text{Thermal } ^{238}\text{U captures}} = \dfrac{\Sigma_{C_1}^{28}\phi_1}{\Sigma_{C_2}^{28}\phi_2}$

^{235}U fission ratio $\quad\quad\quad \delta^{25} = \dfrac{\text{Epithermal } ^{235}\text{U fissions}^{\dagger}}{\text{Thermal } ^{235}\text{U fissions}} = \dfrac{\Sigma_{f_1}^{25}\phi_1}{\Sigma_{f_2}^{25}\phi_2}$

$$\text{(2)}$$

^{238}U fast fission ratio $\quad \delta^{28} = \dfrac{\text{Total fissions in } ^{238}\text{U}}{\text{Total fissions in } ^{235}\text{U}} = \dfrac{\Sigma_{f_1}^{28}\phi_1}{\Sigma_{f_1}^{25}\phi_1 + \Sigma_{f_2}^{25}\phi_2}$

modified conversion ratio $\quad C^* = \dfrac{\text{Total } ^{238}\text{U captures}}{\text{Total } ^{235}\text{U fissions}} = \dfrac{\Sigma_{C_1}^{28}\phi_1 + \Sigma_{C_2}^{28}\phi_2}{\Sigma_{f_1}^{25}\phi_1 + \Sigma_{f_2}^{25}\phi_2}$

† relative to a Cd cut-off defining the group 1 - group 2 break.

Define in addition a new quantity

$$\xi \equiv \frac{\text{Total captures in materials other than } ^{235}\text{U or } ^{238}\text{U}}{\text{Total fission in } ^{235}\text{U}} = \frac{\Sigma_{C_1}^{O}\phi_1 + \Sigma_{C_2}^{O}\phi_2}{\Sigma_{f_1}^{25}\phi_1 + \Sigma_{f_2}^{25}\phi_2} \quad\quad \text{(3)}$$

The (n,2n) reactions can be included in the numerator of ξ as negative capture events. Also define

$$\bar{\nu}_n = \frac{\overline{\nu \Sigma}_{f_n}}{\overline{\Sigma}_{f_n}} \quad \text{and} \quad \bar{\alpha}_n = \frac{\overline{\Sigma}_{c_n}}{\overline{\Sigma}_{f_n}} \quad\quad\quad \text{(4)}$$

Using the definitions in equations 2, 3, and 4 allows equation 1a to be written

$$\left\{ \frac{D_1\phi_1/\phi_2 + D_2}{\Sigma_{f_2}} \right\} B_m{}^2 = (\bar{\nu}_2^{25} - 1) + (\bar{\nu}_1^{25} - 1)\,\delta^{25} - \bar{\alpha}_2^{25} - \bar{\alpha}_1^{25}\delta^{25} +$$

$$\text{(5)}$$

$$(1 + \delta^{25})\left\{ (\bar{\nu}_1^{28} - 1)\delta^{28} - C^* - \xi \right\}$$

The ϕ_1/ϕ_2 flux ratio is eliminated using the group 2 balance equation

$$(\Sigma_{a_2} + \Sigma_{r_{2 \to 1}} + D_2 B_m^2)\phi_2 = \Sigma_{r_{1 \to 2}}\phi_1 \tag{6}$$

Substitution of equation 6 into equation 5 yields the final expression.

$$B_m^2 \left\{ \frac{D_1(\Sigma_{a_2} + \Sigma_{r_{2 \to 1}} + D_2 B_m^2)}{\Sigma_{f_2} \Sigma_{r_{1 \to 2}}} + \frac{D_2}{\Sigma_{f_2}} \right\} = (\bar{\nu}_2^{25} - 1) + (\bar{\nu}_1^{25} - 1)\delta^{25} - \bar{\alpha}_2^{25} - \bar{\alpha}_1^{25}\delta^{25} + \tag{7}$$

$$+ (1 + \delta^{25}) \left\{ (\bar{\nu}_1^{28} - 1)\delta^{28} - C^* - \xi \right\}$$

Equation 7 could be expressed in terms of ρ^{28} rather than C^* using

$$C^* = \frac{\Sigma_{C_2}^{28} \phi_2 (1 + \rho^{28})}{\Sigma_{f_2}^{25} \phi_2 (1 + \delta^{25})} = \frac{C^{28}}{F^{25}} \frac{(1 + \rho^{28})}{(1 + \delta^{25})} \tag{8}$$

C^{28}/F^{25} is the most commonly used symbol[10] for the thermal cross section ratio. Equation 8 can also be rewritten to predict C^{28}/F^{25} as

$$C^{28}/F^{25} = C^* \frac{(1 + \delta^{25})}{(1 + \rho^{28})} = \frac{\Sigma_{C_2}^{28}}{\Sigma_{f_2}^{25}} \tag{8a}$$

This procedure has been suggested as a consistency check[10] on lattices since C^{28}/F^{25} should be calculated by any thermal spectrum code to 1% or better accuracy.

The definition of C^* in equation 8 has been called the "indirect" C^* by Hardy, et al.[3] and the experimentally determined C^* the "direct" C^*. This nomenclature will be used throughout this discussion.

Equation 7 is the desired relationship linking material buckling to measured activation parameters. It is an exact neutron balance equation at $k_{eff} = 1$ and should be satisfied by all transport theory lattice analysis codes even if they are inaccurate.

The quantities in equation 7 fall into three categories:

o   Quantities that are the most difficult to calculate, hence
    are measured. (δ^{25}, δ^{28}, C^* or ρ^{28})

o   Quantities that involve basic nuclear data that may be
    calculated within the uncertainties of the basic data.

    ($\bar{\nu}_1^{25}$, $\bar{\nu}_2^{25}$, $\bar{\nu}_1^{28}$ $\bar{\alpha}_2^{25}$, $\bar{\alpha}_2^{28}$, $\Sigma_{C_2}^{28}/\Sigma_{f_2}^{25}$)

o   Quantities that are truly cell dependent, but which present
    generation lattice physics codes should be able to calculate
    satisfactorily.

    (D_1, D_2, $\Sigma_{r_{1\to2}}$, $\Sigma_{r_{2\to1}}$, Σ_{f_2}, Σ_{C_2}, and ξ)

The parameter ξ is an important one in that it plays an equivalent
role to C^* in equation 1. For H_2O moderated lattices ξ is of magnitude
~ 0.4 and for D_2O moderated lattices ξ is of magnitude ~ 0.03. One
expects, therefore, that H_2O moderated lattices will be much more
sensitive to the ξ parameter.

II. Procedures for Consistency Testing

In one test for consistency, B_m^2 was determined by equation 7
from the measured activation parameters (direct C^*) and compared with
the measured B_m^2. A second test was identical to this except that
the indirect C^* from equation 8 was used in equation 7. Finally,
activation parameters alone were tested for consistency by comparing
C^{28}/F^{25} from equation 8a with the calculated value from RAHABR.

Not all tests were applicable to each set of experiments. In
two experiments, for example, all activation results were reported
but B_m^2 was not. Thus only the third test above was possible.

Details of the procedure are given below.

o   An infinite lattice cell calculation was performed for
    each benchmark lattice tested using the RAHABR[2] lattice
    analysis module and ENDF/B Version IV cross sections.[12]
    RAHABR contains the newly developed resonance capture
    module RRR1D[2] and is an advanced state-of-the-art lattice
    analysis system. The lattice integral parameters and the
    two-group constants required for equation 7 were taken from
    the RAHABR edits at $k_{eff} = 1$ after a buckling search.

o   The full set of integral parameters and the two-group constants
    from the RAHABR calculation were substituted into equation 7.
    Full consistency of the RAHABR calculation was then checked
    if the B_m^2 predicted by equation 7 agreed with the RAHABR

predicted $B_m{}^2$. In every case agreement to at least four significant digits was obtained.

o   The experimental integral parameters (δ^{25}, δ^{28}, and the direct C^*) were then substituted into equation 7 with the other two-group constants generated by RAHABR. The $B_m{}^2$ predicted by equation 7 was then compared to the experimental $B_m{}^2$. Uncertainties assigned to the activation parameters were combined via equation 7 and the usual statistical rules to obtain a minimum uncertainty to assign to the $B_m{}^2$ from equation 7. If a discrepancy existed between the experiment and the $B_m{}^2$ from equation 7, the change required in δ^{25}, δ^{28}, C^*, and ρ^{28} individually to account for the discrepancy was computed to identify high sensitivity parameters.

o C^* was calculated from equation 8 using C^{28}/F^{25} from the RAHABR calculation. This indirect C^* and the other experimental parameters were then substituted into equation 7 with the two-group constants generated by RAHABR. The same comparison of experimental and calculated $B_m{}^2$ and sensitivities to individual parameters were made as in the previous step.

o   The C^{28}/F^{25} ratio predicted from equation 8a was compared with the RAHABR calculated value, which should be accurate to within the cross section uncertainties.

o   The last three previous steps were repeated for as many sets of experimental data as was available for each lattice.

The steps outlined above were incorporated into a computer program (also a JOSHUA module) called BENMRK. This program was able to do all of the above steps with full error analysis for all nineteen lattices in this study in one second of IBM 360/195 CPU time.

III.   Detailed Results of Consistency Testing

        The following gives a detailed presentation of consistency results for each category of lattice.

1.   BAPL One Region H_2O Moderated Critical Lattices[3] (CSEWG Benchmarks TRX-1 and TRX-2)[4]

        These lattices consisted of aluminum clad 0.983 cm dia. U-metal rods enriched to 1.3 w/o ^{235}U, and moderator to fuel volume ratios of 2.35 (TRX-1) and 4.02 (TRX-2). These lattices have been extensively used as benchmarks for cross section analysis of various ENDF/B versions by the Cross Section Evaluation Working Group (CSEWG). The

consistency testing results are summarized in Table I for these lattices. Three reported sets of experimental results, the RAHABR calculated integral parameters, and the results from equation 7 and 8a are shown for both direct and indirect C*. The three sets of experimental results are

1. original reported directly measured parameters in reference 3.

2. CSEWG specified parameters in reference 4.

3. modified parameters in reference 6.

There is very little difference in the three sets of experimental results, and the results derived from equation 7 are very similar. The discrepancies in the experimental B_m^2 and that obtained from equation 7 can be totally accounted for with approximately 5%-10% errors in δ^{25} or δ^{28}, 2%-4% errors in ρ^{28}, or 1%-2% errors in C* or ξ. The discrepancy is more likely due to the errors in resonance region data in ENDF/B-IV combined with systematic errors of 1% or less in all of the measured parameters.

Table I shows significant differences in the direct and indirect C* parameter. The original reported data[3] showed very good agreement beteeen the two C*'s. This discrepancy is due to the thermal cross sections used in the calculation of C^{28}/F^{25} in equation 8 as shown below.

C^{28}/F^{25} VALUES USED IN DIFFERENT C* CALCULATIONS

	Hardy-Ref. 3	Hardy-ENDF/B-IV[11]	RAHABR-ENDF/B-IV
TRX-1	.37620	.37147	.36991
TRX-2	.37314	.36853	.36718
TRX-3	.38505	.38069	.37751
TRX-4	.37122	.36590	.36538

The ENDF/B-IV results of Hardy and RAHABR agree to 1/4% for all of these lattices whereas the C^{28}/F^{25} ratio used in the original experiment was 1-2% higher.

Overall no serious discrepancies are seen in these results. This conclusion is independent of which set of experimental data is chosen.

2. BAPL Two Region H$_2$O Moderated Lattices[3] (CSEWG Benchmarks
    TRX-3 and TRX-4)[4]

     These lattices consisted of aluminum clad 0.983 cm dia. U-metal
rods enriched to 1.3 w/o ^{235}U, and moderator to fuel volume ratios
of 1.0 (TRX-3) and 8.11 (TRX-4). The test lattice was surrounded
by a UO$_2$ driver lattice to achieve criticality. These lattices have
been only occasionally used as benchmarks for cross section analysis
because they lack a reported reactivity parameter (B$_m^2$) for calcula-
tional comparison. Their presence in this report is due to the use
of these benchmarks by CSEWG laboratories. Consistency test results
for these lattices are shown in Table II. Two sets of experimental
results, the RAHABR calculated integral parameters, and the results
from equations 7 and 8a are shown for both the direct and indirect
C*. The two sets of experimental results are

     1. Original experimental parameters from References 3 and 4.

     2. Modified parameters from Reference 6.

     Because no material buckling was reported the RAHABR calculated
result with a 10% uncertainty was used as an experimental B$_m^2$ for
consistency checking purposes. The two sets of experimental data
give almost the same results when put into equations 7 and 8a.
These results do not agree with the RAHABR calculation.

     Hardy[11] provided additional information on these lattices. The
experimental lattice had significant radial leakage between the test
core and the driver core, and the axial flux shape in the test core
deviated significantly from a cosine shape. A two-dimensional trans-
port theory lattice calculation assumes zero radial leakage and a
cosine axial shape to perform its leakage calculation (which is very
close to the uniform one-region critical lattice condition). It is
not at all clear how a two-dimensional calculation, such as RAHABR,
may be used to analyze the leakage conditions, hence reactivity
parameters, in these lattices.

     These lattices are currently being analyzed by Hardy[11] using
3-D Monte Carlo codes.

     In view of the significant problems with analysis of these
lattices they do not appear to be useful benchmark experiments.

3. MIT Sub-critical D$_2$O Moderated Lattices of 1.01 Inch Natural
    U-Metal Rods[5] (CSEWG Benchmarks MIT-1, MIT-2, MIT-3)[4]

     These lattices consisted of aluminum clad 1.01 inch dia. natural
U-metal rods on triangular lattice pitches of 4.5 inches (MIT-1),
5.0 inches (MIT-2), and 5.75 inches (MIT-3).

These lattices have been used extensively by CSEWG for cross section testing purposes with various ENDF/B versions. The consistency testing results for these lattices are summarized in Table III and plotted against lattice pitch in Figure 1. Two sets of experimental results, the RAHABR calculated integral parameters, and the results from equations 7 and 8a are shown for both direct and indirect C*. The two experimental sets are

1. Original experimental parameters from references 4 and 5.

2. Modified parameters from reference 6.

There is too little difference between these two sets of experimental results to be visible in Figure 1.

Figure 1 shows the variation of the various values of B_m^2 and C* with lattice pitch. C* is plotted because these lattices are highly sensitive (as are all D_2O lattices) to variations in C*. To account for the discrepancy in B_m^2 between experiment and equation 1 for the direct C* would require a 100% change in δ^{25} or δ^{28}, a 45% change in ρ^{28}, or an 8% change in C*.

The important feature of Figure 1 is not the shape of the curve but the dispersion of the various curves of B_m^2 or C*. A consistent lattice will have all curves bunched into a narrow band. Obviously these lattices do not meet this criteria.

The experimental B_m^2 appears to disagree with all values derived from equation 7 using different experimental parameters and C*'s. These lattice experiments are inconsistent.

4. SRL Sub-critical D_2O Moderated Lattices of Mark 5R Assemblies[7]

These lattices consisted of assemblies containing two annular coaxial aluminum clad fuel tubes of U-metal enriched to 0.86 w/o ^{235}U and triangular lattice pitches of 5.5, 6.0, 7.0, and 8.0 inches.

These lattices were proposed as CSEWG thermal benchmark lattices and have been used for cross section analysis purposes at SRL. The consistency testing results for these lattices are summarized in Table IV and plotted against lattice pitch in Figure 2. The original experimental data, the RAHABR calculations, and the results of equations 7 and 8a are shown for direct and indirect C*.

Figure 2 shows the variation of the different B_m^2 values and C* values with lattice pitch. Note that Figure 2 has a very compact scale so that small dispersions of the various curves are very significant.

The $B_m{}^2$ for direct and indirect C^* values from equation 7 agree well with each other but disagree strongly with experimental bucklings.

The RAHABR calculation disagrees with experiment for both $B_m{}^2$ and C^* (direct and indirect). The only obvious trend is that the experimental $B_m{}^2$ is always larger than calculation or prediction by equation 7.

These lattices are internally inconsistent and are not recommended as thermal benchmark lattices.

5. SRL Sub-critical D$_2$O Moderated Lattices of Mark 15 Assemblies[7]

These lattices consisted of assemblies containing two annular coaxial aluminum clad fuel tubes of U-metal enriched to 1.10 w/o ^{235}U and triangular lattice pitches of 6.35, 7.0, 8.08, and 9.25 inches.

These lattices were proposed as CSEWG thermal benchmark lattices and have been used for cross section analysis and data adjustment at SRL. The consistency testing results for these lattices are summarized in Table V and plotted against lattice pitch in Figure 3. The experimental results, the RAHABR calculated results, and the results from equations 7 and 8a are shown for direct and indirect C^* values.

The results in Table V and Figure 3 follow exactly the same pattern as the Mark 5R results. All comments for the Mark 5R are applicable to these lattices.

These lattices are internally inconsistent and are not recommended as thermal benchmark lattices.

6. MIT Sub-critical D$_2$O Moderated Lattices of 0.387 Inch Dia. U-Metal Rods[8]

These lattices consisted of aluminum clad 0.387 inch dia. U-metal rods enriched to 0.947 w/o ^{235}U and traingular lattice pitches of 1.5, 2.25, and 3.0 inches. These lattices were a later series of measurements in the MIT Heavy Water Lattice Project. The CSEWG thermal benchmark lattices MIT-1, MIT-2, and MIT-3 were the first lattices measured in the project. The consistency testing results for these lattices are summarized in Table VI and plotted against lattice pitch in Figure 4. The experimental results, the RAHABR calculated results, and the results from equations 7 and 8a are shown for direct and indirect C^* values.

Figure 4 shows excellent agreement for B_m^2 and C^* between the RAHABR calculation and the values predicted by equations 7 and 8. The experimental B_m^2 is in disagreement with both calculation and equation 7. Activation parameters show good agreement with the RAHABR calculation.

Table VII summarizes the experimental data[8] for the buckling in these lattices. Aside from the strange uncertainties quoted when averaging the bare and Cd covered foil data, the bucklings appear to have an energy dependence. The material bucklings derived from the bare foil and Cd covered foil data separately show differences of considerably greater magnitude than the stated uncertainty.

To utilize these lattices as thermal benchmarks will require these material buckling discrepancies to be resolved.

7. CRNL D$_2$O Moderated Critical Lattices of ZEEP Assemblies[9]

These lattices consisted of aluminum clad 1.28 inch dia. natural U-metal rods on triangular lattice pitches from 4.75 to 8.66 inches. Data for these lattices were supplied by W. H. Walker[9] of Chalk River National Laboratory. Material buckling measurements were made at several lattice pitches, but only at 7.875 inch (20.0 cm) triangular pitch were B_m^2, δ^{28}, and the conversion ratio CR measured. The conversion ratio CR is defined by

$$CR \equiv \frac{\text{Total captures in } ^{238}U}{\text{Total absorptions in } ^{235}U}$$

This value was converted to a C^* value by assuming the CR/C^* value calculated from RAHABR was correct. Since δ^{25} was not measured, the value calculated by RAHABR was used.

Consistenty testing results for this lattice are shown in Table VIII. The B_m^2 from equation 7 and experiment agree very well. The RAHABR calculation is in good agreement with equation 7 and experiment as well.

The remaining experiments in this series should provide a good test for calculating k_{eff}.

IV. General Conclusions

The material bucklings for all of the above lattices obtained from 1) experiment, 2) from equation 7 with the direct C^*, 3) from equation 7 with the indirect C^*, and 4) from RAHABR calculation are shown in Table IX and Table X which summarize most of the consistencies and inconsistencies observed in this work.

The overall picture presented by these data can be summarized in the following statements:

o   Only a few of the CSEWG thermal benchmark lattice experiments that are used routinely for cross section testing are in fact generally adequate for that purpose (only TRX-1 and TRX-2).

o   Sub-critical D_2O lattice experiments show systematic inconsistencies.

o   For lattices that are internally consistent the RAHABR module shows good calculational agreement to the experiment. Further the source of most of the disagreement is known from present knowledge of the defects in ENDF/B Version IV cross sections.

The first two of these are very distressing, for good thermal benchmark lattices are needed to assess the accuracy of computational methods and for cross section testing. The sub-critical experiment discrepancies are of great concern because all of the clean and complete D_2O moderated benchmark lattice experiments are eliminated from consideration.

There appears to be no reason to suspect the experimental measuring methods in any of these lattices. As seen in the TRX and MIT series of lattices the corrections of Sher[6] have only a minor effect on the results. The experimental methods were pretty much the same in all of the experiments analyzed here, the differences being mainly in the methods of making foil corrections by the different experimenters.

V.   Speculation on Sub-critical D_2O Lattice Experiments

Because experimental techniques do not affect the consistency testing results, it would appear that the conceptual design of the sub-critical experiment may be in question. Two statements summarize the consistency testing results for the sub-critical experiments:

o   The experimental material bucklings are consistently higher than those predicted by equation 7 by amounts that are outside the range of assigned uncertainties.

o   Activation parameters are sometimes higher and sometimes lower in value than RAHABR calculated values, but are higher or lower as a whole set rather than having a random variation.

The first of these statements may arise from the concept of the sub-critical experiment. The experimenter may be measuring a geometrical buckling that is not the material buckling.

The second of these statements is more interesting in that it leads to some enlightening speculation. First, two assumptions are made.

1. The entire discrepancy of all the activation parameters arises from a shift in the neutron spectrum from that which would be observed in a critical experiment.

2. Since the RAHABR calculation is in good agreement with the consistent lattices, assume it calculates the correct results for the inconsistent lattices for a critical experiment neutron spectrum.

Identify now the RAHABR calculated activation parameter by the normal symbol and the measured parameter by the normal symbol with a bar over it (e.g., ρ^{28} and $\bar{\rho}^{28}$, etc.). Let ε represent a fractional excess thermal leakage so that the relations between calculated parameters and experimental parameters are:

$$\bar{\rho}^{28} = \frac{\rho^{28}}{1-\varepsilon} = \frac{\text{Epithermal captures in } ^{238}U}{\text{Thermal captures in } ^{238}U \times (1-\varepsilon)}$$

$$\bar{\delta}^{25} = \frac{\delta^{25}}{1-\varepsilon} = \frac{\text{Epithermal fissions in } ^{235}U}{\text{Thermal fissions in } ^{235}U \times (1-\varepsilon)} \tag{9}$$

$$\bar{C}^{*} = \frac{\text{Epithermal captures in } ^{238}U + \text{Thermal captures in } ^{238}U \times (1-\varepsilon)}{\text{Epithermal fissions in } ^{235}U + \text{Thermal fissions in } ^{235}U \times (1-\varepsilon)}$$

$$\bar{\delta}^{28} = \frac{\text{Epithermal fissions in } ^{238}U}{\text{Epithermal fissions in } ^{235}U + \text{Thermal fissions in } ^{235}U \times (1-\varepsilon)}$$

Solving for ε then yields

Parameter	Equation
ρ^{28}	$\varepsilon = \dfrac{\bar{\rho}^{28} - \rho^{28}}{\bar{\rho}^{28}}$
δ^{25}	$\varepsilon = \dfrac{\bar{\delta}^{25} - \delta^{25}}{\bar{\delta}^{25}}$
C^{*}	$\varepsilon = \dfrac{(\bar{C}^{*} - C^{*})TF^{25}}{\bar{C}^{*}F^{25} - C^{28}}$

$$\tag{10}$$

Parameter	Equation
δ^{28}	$\varepsilon = \dfrac{(\bar{\delta}^{28} - \delta^{28})(\bar{\delta}^{25} - \delta^{25})}{\bar{\delta}^{28}}$

where:

$TF^{25} \equiv$ Total fissions in ^{235}U

$F^{25} \equiv$ Thermal fissions in ^{235}U

$C^{28} \equiv$ Thermal captures in ^{238}U

In equations 10 a positive ε corresponds to a more epithermal spectrum than RAHABR calculates while a negative ε corresponds to a more thermal spectrum than RAHABR.

In Table XI equations 10 have been applied to the two sets of MIT sub-critical lattices and the two sets of SRL sub-critical lattices. This analysis indicates that spectral shifts of less than 10% could produce all of the discrepancies in activation parameters between experiment and calculation. The important factor is that when calculation and experiment disagree, all spectral shifts determined from all parameters go in the same direction. When the calculation and experiment are in good agreement (MIT 0.387" rods) the shifts are not as consistent in direction.

These data suggest that a measurement region with constant spectrum existed in these sub-critical experiments with a fast to thermal flux ratio (relative to the Cd cut-off) not the same as would have existed in a critical experiment using the same assemblies.

If this is true, a mechanism must exist to explain these spectral shifts; however, none has been found to date. A possible factor that might play a role is the consistently high material buckling, but it seems unlikely due to the positive and negative shifts seen in Table XI.

The only way to test this spectral shift hypothesis is to obtain good spectral ratio data for a lattice measured in both a critical and a sub-critical experiment. Simple radial and axial traverses of Cd covered and bare foils would be adequate. The upcoming measurements of the Mark 15 in the LTR critical facility can offer this opportunity if one lattice can be re-measured in the sub-critical facility.

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TABLE I

CONSISTENCY TESTING RESULTS FOR
BAPL ONE REGION CRITICALS (H_2O CRITICALS)

	TRX-1				TRX-2			
	EXPT 1[3]	EXPT 2[4]	EXPT 3[6]	CALC	EXPT 1[3]	EXPT 2[4]	EXPT 3[6]	CALC
B_m^2 (m^{-2})	57.00±1.00	57.00±1.00	57.00±1.00	56.09	54.69±.36	54.69±.36	54.69±.36	53.84
ρ^{28}	1.311±.02	1.311±.02	1.320±.021	1.3389	.830±.015	.830±.015	.837±.016	.83829
δ^{25}	.0981±.001	.0981±.001	.0987±.001	.093657	.0608±.0007	.0608±.0007	.0614±.0008	.057629
δ^{28}	.0914±.002	.0914±.002	.0945±.008	.092623	.0667±.002	.0667±.002	.0693±.0035	.064200
C^* (direct)	.799±.008	.792±.008	.797±.008	.79110	.648±.006	.646±.002	.647±.006	.63821
k_{eff} at Expt. B_m^2	1.0	1.0	1.0	.99505	1.0	1.0	1.0	.99663
Direct C^*								
B_m^2 (2 Grp)	54.66±1.25	55.74±1.25	55.84±1.55	56.09	53.05±1.06	53.39±.64	53.96±1.32	53.84
C^{28}/F^{25}	.379±.005	.376±.005	.377±.005	.36991	.376±.005	.374±.003	.374±.005	.36718
Indirect C^*								
B_m^2 (2 Grp)	57.51±1.10	57.62±1.10	58.06±1.46	56.09	55.20±.96	55.25±.96	55.66±1.28	53.84
C^{28}/F^{25}	.36991	.36991	.36991	.36991	.36718	.36718	.36718	.36718
C^* (indirect)	.778±.007	.778±.007	.781±.007	.79110	.633±.005	.633±.005	.636±.006	.63821

TABLE II

CONSISTENCY TESTING RESULTS FOR
BAPL TWO REGION CRITICALS (H_2O CRITICALS)

	TRX-3			TRX-4		
	EXPT 1 [5]	EXPT 2 [6]	CALC	EXPT 1 [5]	EXPT 2 [6]	CALC
B_m^2 (m^{-2})	19.65 ± 2.00^a	19.65 ± 2.00^a	19.67	$5.13\pm.50^a$	$5.13\pm.50^a$	5.13
ρ^{28}	$3.01\pm.05$	$3.03\pm.05$	2.9674	$.466\pm.01$	$.481\pm.011$	.48203
δ^{25}	$.230\pm.003$	$.231\pm.003$	.21934	$.0352\pm.0004$	$.0358\pm.0005$	.033731
δ^{28}	$.163\pm.004$	$.167\pm.008$	.17227	$.0452\pm.0007$	$.0482\pm.002$	.054336
C* (Direct)	$1.255\pm.011$	$1.255\pm.011$	1.2283	$.526\pm.004$	$.531\pm.004$	.52383
k_{eff} at Expt. B_m^2	1.0	1.0	1.0	1.0	1.0	1.0
Direct C*						
B_m^2 (2 Grp)	15.12 ± 1.43	15.87 ± 1.94	19.67	$2.54\pm.58$	$2.59\pm.74$	5.13
C^{28}/F^{25}	$.385\pm.006$	$.383\pm.006$	.37751	$.371\pm.004$	$.371\pm.004$	.36538
Indirect C*						
B_m^2 (2 Grp)	17.66 ± 1.85	17.88 ± 2.27	19.67	$3.70\pm.52$	$3.75\pm.73$	5.13
C^{28}/F^{25}	.37751	.37751	.37751	.36538	.36538	.36538
C* (indirect)	$1.231\pm.016$	$1.236\pm.016$		$.517\pm.004$	$.522\pm.004$	

a based on calculated B_m^2 with 10% uncertainty.

TABLE III

CONSISTENCY TESTING RESULTS FOR
MIT 1.01" DIA. NATURAL U-METAL RODS (D$_2$O SUB-CRITICALS)

	MIT-1			MIT-2			MIT-3		
	EXPT 1[5]	EXPT 2[6]	CALC	EXPT 1[5]	EXPT 2[6]	CALC	EXPT 1[5]	EXPT 2[6]	CALC
B_m^2 (m^{-2})	8.48±.10	8.48±.10	8.01	8.65±.10	8.65±.10	8.18	8.15±.08	8.15±.08	7.81
ρ^{28}	.498±.008	.502±.01	.51323	.394±.002	.400±.004	.42262	.305±.004	.313±.005	.32372
δ^{25}	.0447±.0019	.0469±.0019	.047291	.0310±.003	.0335±.003	.038874	.0248±.001	.0265±.0011	.029824
δ^{28}	.0597±.002	.0588±.003	.061535	.0596±.0017	.0587±.003	.06001	.0583±.001	.0575±.003	.057826
C* (direct)	1.017±.023	1.017±.023	.96084	.948±.020	.948±.020	.90830	.859±.016	.859±.016	.85018
k_{eff} at Expt. B_m	1.0	1.0	.99105	1.0	1.0	.98919	1.0	1.0	.99088
Direct C*									
B_m^2 (2 Grp)	6.72±.53	6.69±.54	8.01	7.36±.41	7.33±.42	8.18	7.66±.28	7.65±.29	7.81
C^{28}/F^{25}	.709±.016	.709±.016	.66499	.701±.015	.700±.015	.66329	.675±.013	.672±.013	.66143
Indirect C*									
B_m^2 (2 Grp)	8.08±.15	8.04±.20	8.01	8.34±.11	8.28±.15	8.18	7.94±.06	7.86±.11	7.81
C^{28}/F^{25}	.66499	.66499	.66499	.66329	.66329	.66329	.66143	.66143	.66143
C* (indirect)	.954±.005	.954±.007	.96084	.897±.003	.899±.004	.90830	.842±.003	.846±.003	.85018

TABLE IV

CONSISTENCY TESTING RESULTS FOR
SRL MARK 5R (TYPE I) LATTICES[7] (D_2O SUB-CRITICALS)

	5.5" Pitch		6.0" Pitch		7.0" Pitch		8.0" Pitch	
	EXPT	CALC	EXPT	CALC	EXPT	CALC	EXPT	CALC
B_m^2 (m^{-2})	-4.92±.36	-5.93	0.42±.36	-0.545	6.77±.29	5.72	9.20±.22	8.50
ρ^{28}	2.472±.018	2.2109	1.942±.040	1.7582	1.323±.025	1.2130	.949±.030	.90904
δ^{25}	0.222±.004	.20756	.173±.003	.16535	.120±.002	.11435	.0874±.0015	.085939
δ^{28}	0.107±.005	.10121	.102±.005	.093709	.0921±.004	.084223	.0876±.004	.080153
C* (Direct)	1.649±.020	1.5328	1.449±.020	1.3542	1.158±.015	1.1247	1.024±.013	.98919
k_{eff} at Expt. B_m^2	1.0	.98654	1.0	.98660	1.0	.98401	1.0	.98812
Direct C*								
B_m^2 (2 Grp)	-9.49±.72	-5.93	-3.04±.68	-0.545	5.19±.47	5.72	7.98±.38	8.50
C^{28}/F^{25}	.580±.008	.57646	.578±.011	.57214	.558±.009	.56637	.571±.011	.56269
Indirect C*								
B_m^2 (2 Grp)	-9.13±.46	-5.93	-2.61±.68	-0.545	4.73±.42	5.72	8.36±.43	8.50
C^{28}/F^{25}	.57646	.57646	.57214	.57214	.56637	.56637	.56269	.56269
C* (indirect)	1.638±.010	1.5328	1.435±.020	1.3542	1.175±.013	1.1247	1.009±.016	.98919

TABLE V

CONSISTENCY TESTING RESULTS FOR
SRL MARK 15 (TYPE II) LATTICES[7] (D$_2$O SUB-CRITICALS)

	6.35" Pitch		7.0" Pitch		8.08" Pitch		9.25" Pitch	
	EXPT	CALC	EXPT	CALC	EXPT	CALC	EXPT	CALC
B_m^2 (m^{-2})	2.78±.30	2.18	7.70±.30	7.14	11.82±.22	11.52	13.18±.22	13.17
ρ^{28}	2.507±.065	2.3836	1.979±.030	1.8567	1.404±.022	1.3292	1.073±.026	.99912
δ^{25}	0.220±.004	0.21181	0.170±.003	.16561	.123±.002	.11932	.0923±.0015	.090627
δ^{28}	.0962±.004	.085704	.0850±.004	.079145	.080±.004	.073421	.0805±.004	.070258
C* (Direct)	1.305±.018	1.2563	1.135±.015	1.0939	.959±.04	.92016	.847±.011	.80536
k_{eff} at Expt. B_m^2	1.0	.99210	1.0	.99220	1.0	.99541	1.0	.99974
Direct C*								
B_m^2 (2 Grp)	1.18±.66	2.18	6.19±.54	7.14	10.79±.46	11.52	12.61±.34	13.17
C^{28}/F^{25}	.453±.011	.44994	.446±.0075	.44632	.448±.008	.44219	.446±.008	.43937
Indirect C*								
B_m^2 (2 Grp)	1.56±.85	2.18	6.15±.45	7.14	11.14±.34	11.52	12.93±.33	13.17
C^{28}/F^{25}	.44994	.44994	.44632	.44632	.44219	.44219	.43937	.43937
C* (indirect)	1.293±.024	1.2563	1.136±.012	1.0939	.947±.009	.92016	.834±.011	.80536

TABLE VI

CONSISTENCY TESTING RESULTS FOR
MIT 0.387" DIA. U-METAL RODS[8] ENRICHED TO .947 WT % ^{235}U (D$_2$O SUB-CRITICAL)

	1.5 inch Pitch		2.25 inch Pitch		3.0 inch Pitch	
	EXPT[8]	CALC	EXPT[8]	CALC	EXPT[8]	CALC
B_m^2 (m^{-2})	9.55±.10	7.55	12.44±.11	11.73	10.41±.14	10.08
ρ^{28}	1.155±.001	1.16808	.525±.002	.52469	.317±.002	.30572
δ^{25}	.0865±.0016	.082602	.0371±.0012	.036587	.0222±.0024	.021410
δ^{28}	.0459±.0013	.041659	.0326±.001	.030053	.0291±.0018	.026596
C* (Direct)	1.007±.008	1.0164	.740±.007	.73578	.647±.002	.63598
k_{eff} at Expt. B_m^2	1.0	.97012	1.0	.98595	1.0	.99141
Direct C*						
B_m^2 (2 Grp)	8.05±.26	7.55	11.74±.17	11.73	9.97±.08	10.08
C^{28}/F^{25}	.507±.004	.50762	.503±.005	.50024	.502±.002	.49750
Indirect C*						
B_m^2 (2 Grp)	8.06±.10	7.55	11.83±.06	11.73	10.07±.08	10.08
C^{28}/F^{25}	.50762	.50762	.50024	.50024	.49750	.49750
C* (indirect)	1.007±.002	1.0164	.736±.001	.73578	.641±.002	.63598

TABLE VII

EXPERIMENTAL BUCKLINGS FOR MIT 0.387" U-METAL RODS
(units are m^{-2})

	1.5" Pitch	2.25" Pitch	3.0" Pitch
Bare Foil Radial Buckling	23.78±.08	24.21±.10	24.45±.09
Cd Covered Foil Radial Buckling	23.48±.05	23.94±.06	23.86±.15
Quoted Radial Buckling	23.64±.05	24.06±.07	24.13±.11
Bare Foil Axial Buckling	13.94±.07	11.46±.09	13.62±.13
Cd Covered Foil Axial Buckling	14.18±.12	11.79±.10	13.82±.06
Quoted Axial Buckling	14.09±.09	11.62±.08	13.72±.08
Quoted B_m^2	9.55±.10	12.44±.11	10.41±.14
B_m^2 from Cd Covered Foils Only	9.30±.10	12.15±.11	10.04±.14
B_m^2 from Bare Foils Only	9.84±.10	12.75±.11	10.83±.10

TABLE VIII

CONSISTENCY TESTING RESULTS FOR CRNL ZEEP LATTICE (D_2O CRITICAL)

Parameter	20 Cm Pitch	
	EXPT[9]	CALC
$B_m{}^2$ (m^{-2})	6.95±.06	6.822
ρ^{28}		.27006
δ^{25}		.026078
δ^{28}	.0676±.0014	.064740
C* (Direct)	.82929±.0025	.82909
CR (Direct)[a]	.7048±.0021	.70463
k_{eff}	1.0	.99631
Direct C*		
$B_{\bar{m}}{}^2$ (2 Grp)	6.89±.05	6.82
C^{28}/F^{25}	.670±.006	.66982

$$^a\ CR \equiv \frac{\text{Total captures in } ^{238}U}{\text{Total absorptions in } ^{235}U}$$

TABLE IX

MATERIAL BUCKLINGS FROM CALCULATION AND
EXPERIMENT FOR CRITICAL LATTICE EXPERIMENTS

Lattice	Pitch	Reference	$B_m^2 (m^{-2})$			
			Experiment	Eq. 7 (Direct C*)	Eq. 7 (Indirect C*)	RAHABR
BAPL TRX-1	1.806 cm	3	57.00±1.00	54.66±1.25	57.51±1.10	56.09
		4	57.00±1.00	55.74±1.25	57.62±1.10	56.09
		6	57.00±1.00	55.84±1.55	58.06±1.46	56.09
BAPL TRX-2	2.174 cm	3	54.69±.36	53.05±1.06	55.20±.96	53.84
		4	54.69±.36	53.39±.64	55.25±.96	53.84
		6	54.69±.36	53.96±1.32	55.66±1.28	53.84
CRNL ZEEP	20.0 cm	9	6.95±.06	6.89±.05		6.82

TABLE X

MATERIAL BUCKLINGS FROM CALCULATION AND
EXPERIMENT FOR SUB-CRITICAL LATTICE EXPERIMENTS

Lattice	Pitch	Reference	$B_m^2 (m^{-2})$			
			Experiment	Eq. 7 (Direct C*)	Eq. 7 (Indirect C*)	RAHABR
MIT-1	4.5"	5	8.48±.10	6.72±.53	8.08±.15	8.01
		6	8.48±.10	6.69±.54	8.04±.20	8.01
MIT-2	4.0"	5	8.65±.10	7.36±.41	8.34±.11	8.18
		6	8.65±.10	7.33±.42	8.28±.15	8.18
MIT-3	5.75"	5	8.15±.08	7.66±.28	7.94±.06	7.81
		6	8.15±.08	7.65±.29	7.86±.11	7.81
MIT 0.387" Rods	1.5"	8	9.55±.10	8.05±.26	8.06±.10	7.55
	2.25"	8	12.44±.11	11.74±.17	11.83±.06	11.73
	3.0"	8	10.41±.14	9.97±.08	10.07±.08	10.08
SRL Mark 5R	5.5"	7	-4.92±.36	-9.49±.72	-9.13±.46	-5.93
	6.0"	7	0.42±.36	-3.04±.68	-2.61±.68	-0.55
	7.0"	7	6.77±.29	5.19±.47	4.73±.42	5.72
	8.0"	7	9.20±.22	7.98±.38	8.36±.43	8.50
SRL Mark 15	6.35"	7	2.78±.20	1.18±.66	1.56±.85	2.18
	7.00"	7	7.70±.30	6.19±.54	6.15±.45	7.14
	8.08"	7	11.82±.22	10.79±.46	11.14±.34	11.52
	9.25"	7	13.18±.22	12.61±.34	12.93±.33	13.17

TABLE XI

INCREASED THERMAL LEAKAGE REQUIRED TO
MAKE UP EXPERIMENT-CALCULATION DISCREPANCY

Mark 5R Lattices

Pitch Parameter	5.5"	6.0"	7.0"	8.0"		Avg
ρ^{28}	10.5%	9.5%	8.3%	4.25%		8.75%
δ^{25}	6.5%	4.5%	5.0%	2.0%		4.5%
C* (direct)	13.0%	12.5%	6.2%	8.25%		10.0%
C* (indirect)	12.0%	10.9%	9.2%	4.75%		9.2%
					Avg	8.1%

Mark 15 Lattices

Pitch Parameter	6.35"	7.0"	8.08"	9.25"		Avg
ρ^{28}	5.0%	6.0%	5.0%	7.0%		5.75%
δ^{25}	3.5%	2.5%	3.25%	2.0%		3.0%
C* (direct)	7.0%	7.0%	8.4%	11.25%		8.5%
C* (indirect)	5.3%	7.1%	6.0%	7.9%		6.6%
					Avg	6.0%

MIT 1.01" Dia. Rod Lattices

Lattice Parameter	MIT-1	MIT-2	MIT-3		Avg
ρ^{28}	-2.2%	-5.7%	-3.4%		-3.8%
δ^{25}	-1.0%	-16.0%	-12.5%		-9.8%
C* (direct)	13.1%	14.5%	4.6%		10.7%
C* (indirect)	-2.5%	-4.1%	-2.3%		-3.0%
				Avg*	-5.5%

MIT 0.387" Dia. Rod Lattices

Pitch Parameter	1.5"	2.25"	3.0"		Avg
ρ^{28}	-1.1%	6.05%	3.6%		.85%
δ^{25}	4.5%	1.4%	3.6%		3.2%
C* (direct)	-2.0%	1.8%	7.5%		2.4%
C* (indirect)	-2.0%	0.1%	3.6%		.6%
				Avg	1.75%

* - C* (direct) omitted in average.

FIGURE 1

MIT 1,01" NATURAL URANIUM ROD LATTICES (D$_2$O SUBCRITICALS)
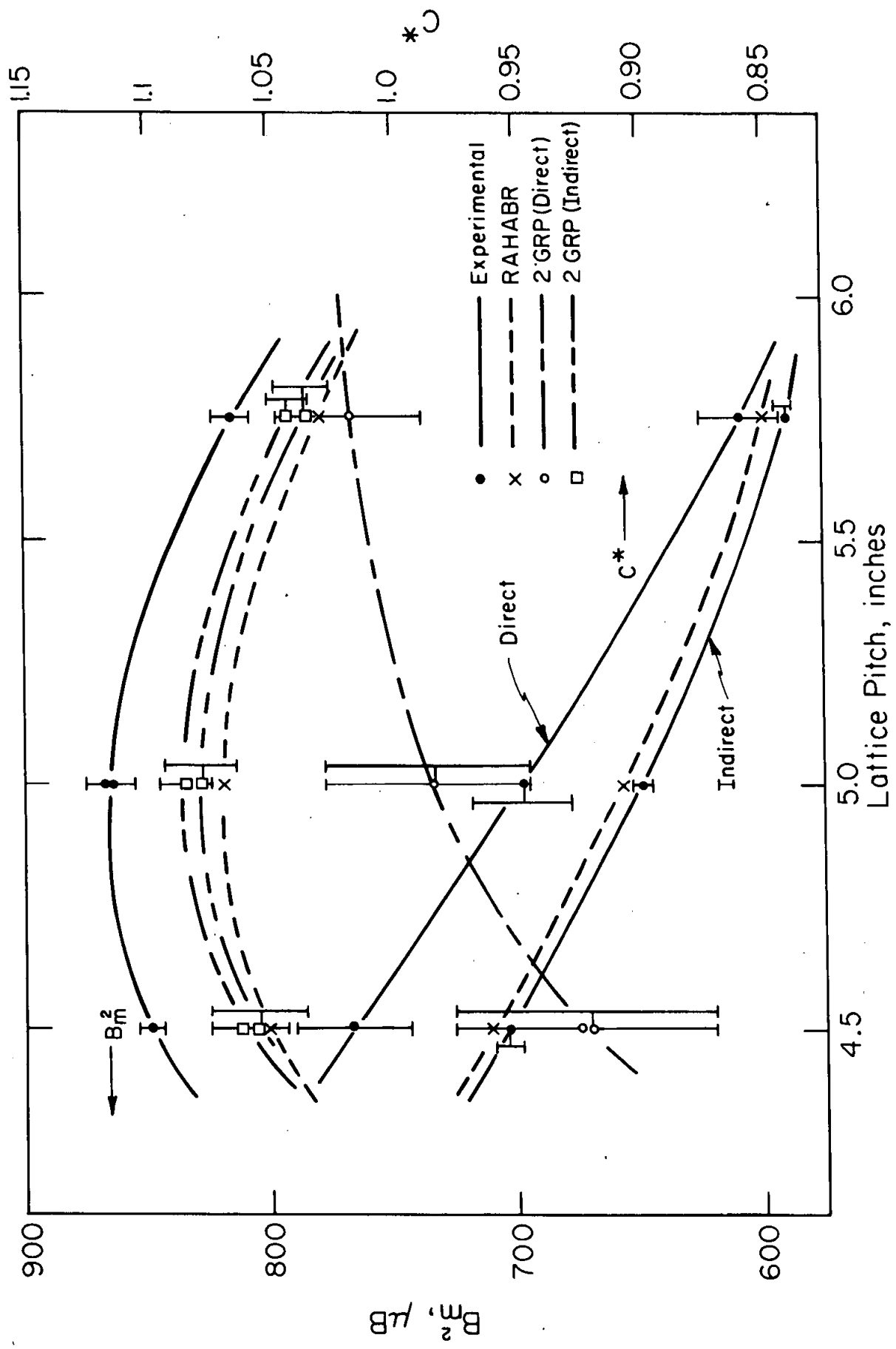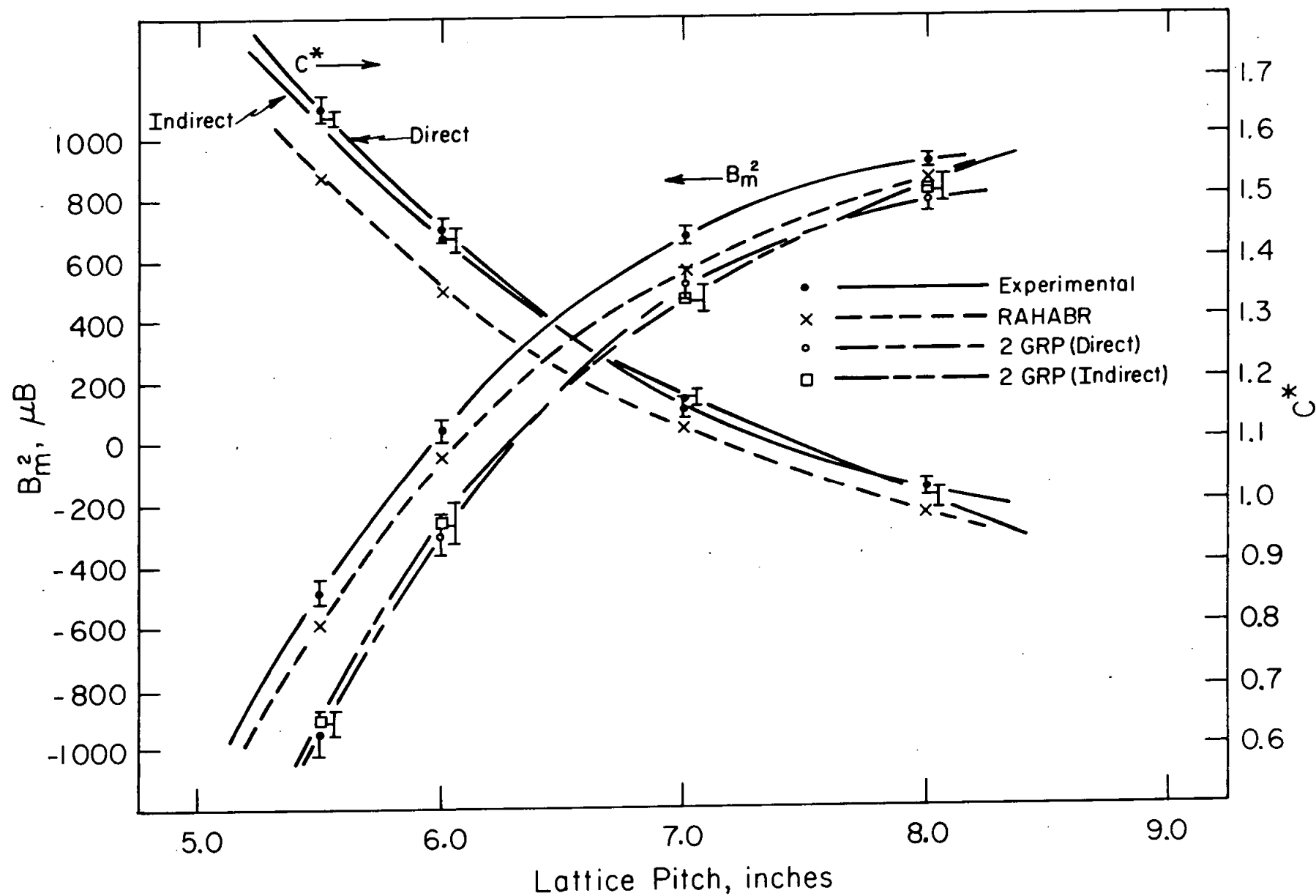
(MIT-1, MIT-2, MIT-3)

MARK 5R (TYPE I) LATTICES (D$_2$O SUBCRITICALS)

P. L. ROGGENKAMP

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FIGURE 3

MARK 15 (TYPE 11) LATTICES (D₂O SUBCRITICALS)
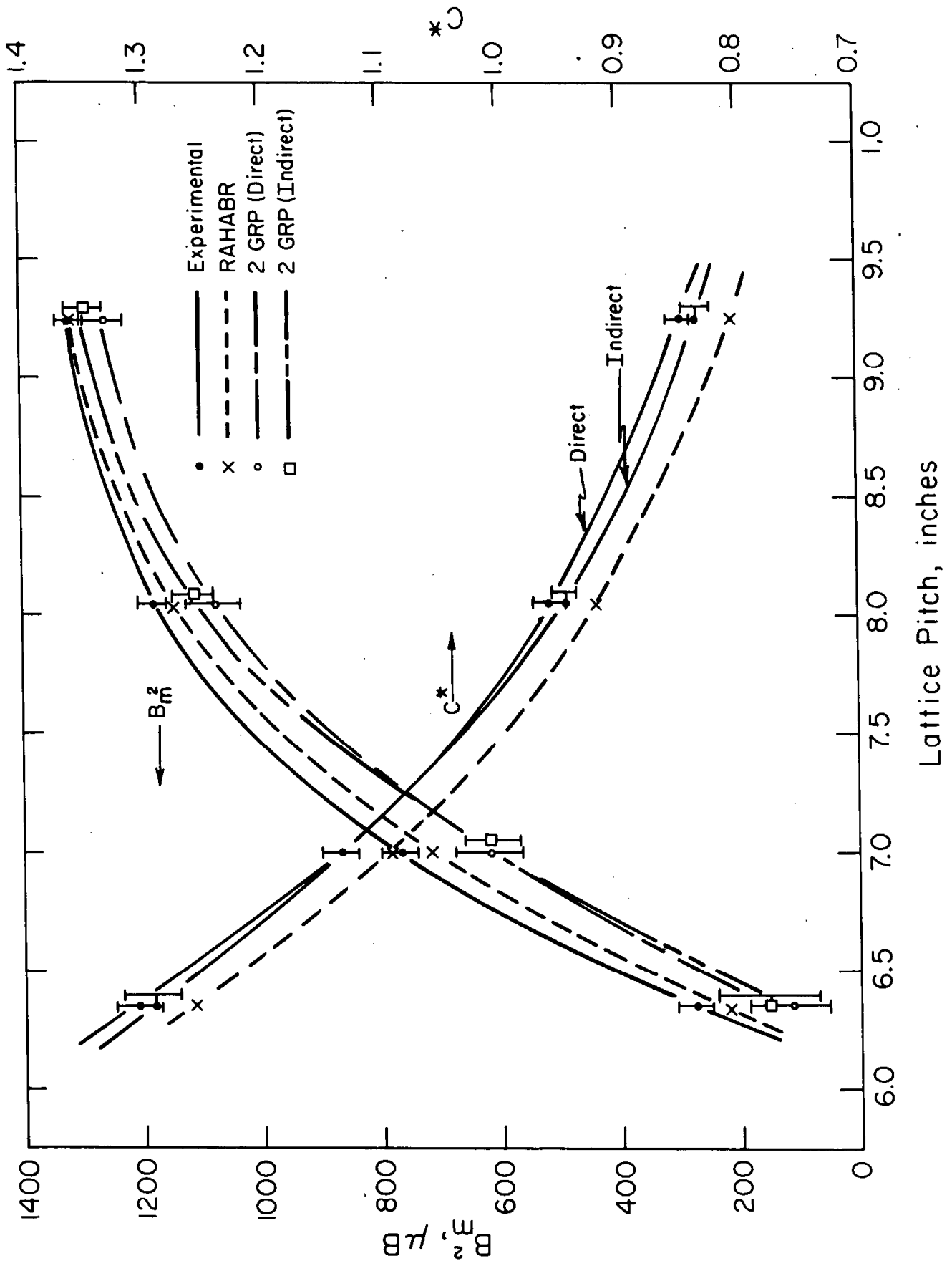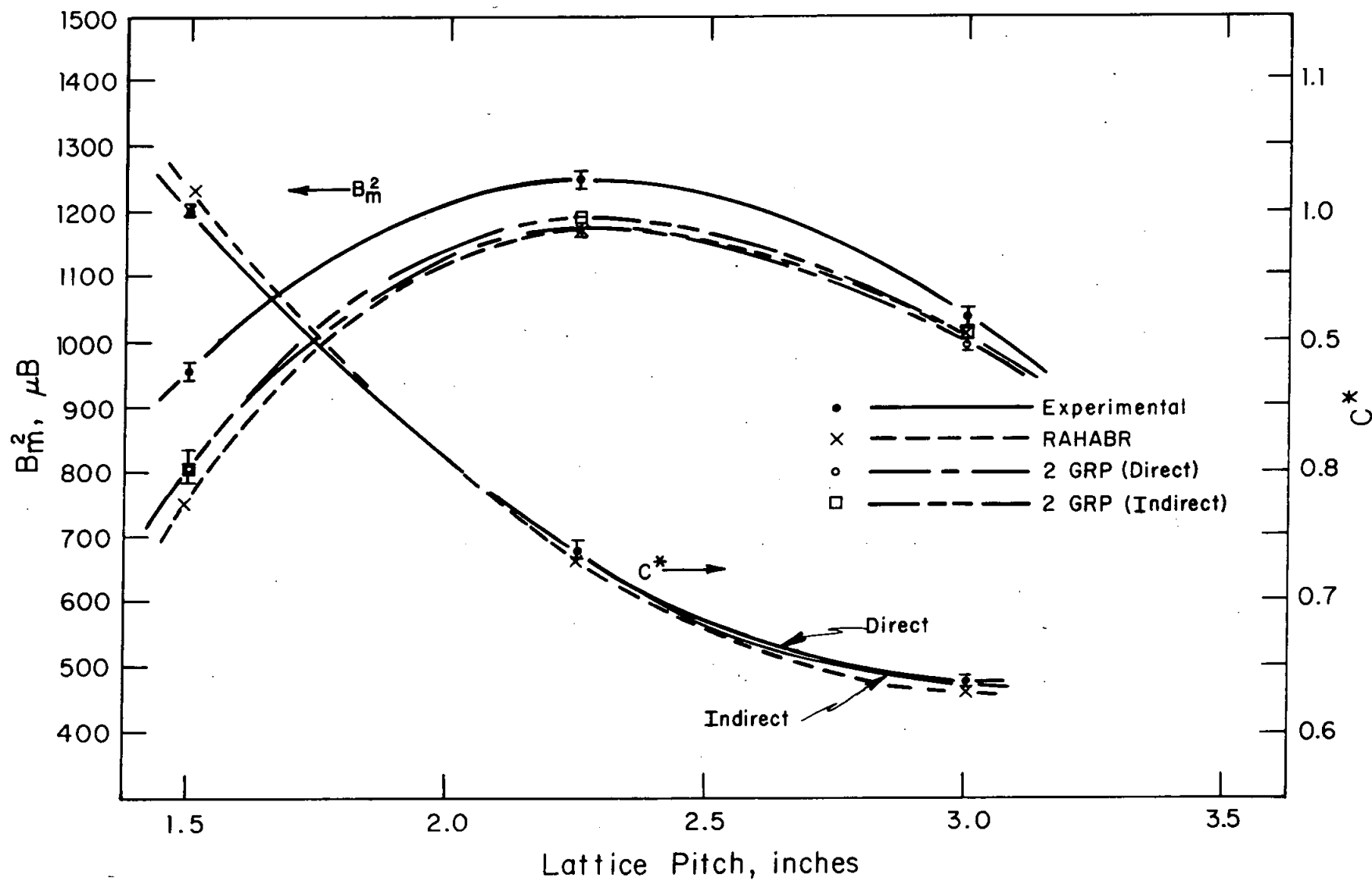
FIGURE 4

MIT 0.387" RODS ENRICHED TO 0.947 W/O ^{235}U (D$_2$O SUBCRITICALS)

APPENDIX A

TWO GROUP PARAMETERS USED IN CONSISTENCY TESTING

The two group parameters for each lattice as calculated by the RAHABR[2] Lattice Analysis Module are tabulated in the following tables.

TABLE A-I

TWO GROUP DATA FOR BAPL LATTICES (RAHABR)

Lattice Constant	TRX-1	TRX-2	TRX-3	TRX-4
$\bar{\alpha}_1^{25}$	.43905	.43915	.43384	.42769
$\bar{\alpha}_2^{25}$	.17223	.17124	.17477	.17047
$\bar{\nu}_1^{25}$	2.4356	2.4369	2.4345	2.4439
$\bar{\nu}_2^{25}$	2.4188	2.4188	2.4188	2.4188
$\bar{\nu}_1^{28}$	2.8135	2.8174	2.8046	2.8223
D_1	1.1316	1.1643	1.0840	1.2113
D_2	0.25741	0.21408	0.36526	0.17966
Σ_{f_2}	.056812	.039696	.079300	.021033
Σ_{a_2}	.099269	.075148	.13068	.048780
$\Sigma_{r_{1\to2}}$	.027424	.034260	.015956	.041538
$\Sigma_{r_{2\to1}}$	3.5725×10^{-5}	2.7370×10^{-5}	4.5336×10^{-5}	1.7734×10^{-5}
ξ	.20357	.35245	.093772	.77809
C^{28}/F^{25}	.36991	.36719	.37751	.36538

TABLE A-II

TWO GROUP DATA FOR MIT LATTICES (RAHABR)

Lattice Constant	MIT-1	MIT-2	MIT-3	1.5" Pitch	2.25" Pitch	3.0" Pitch
$\bar{\alpha}_1{}^{25}$	.44030	.43627	.42940	.46514	.46067	.45283
$\bar{\alpha}_2{}^{25}$	.17141	.17105	.17066	.17282	.17098	.17030
$\bar{\nu}_1{}^{25}$	2.4305	2.4325	2.4357	2.4250	2.4284	2.4326
$\bar{\nu}_2{}^{25}$	2.4188	2.4188	2.4188	2.4188	2.4188	2.4188
$\bar{\nu}_1{}^{28}$	2.8086	2.8078	2.8072	2.8174	2.8158	2.8147
D_1	1.2485	1.2582	1.2665	1.2497	1.2774	1.2880
D_2	.82193	.81834	.81305	.84206	.81604	.80373
Σ_{f_2}	.0041025	.0033067	.0024744	.0088349	.0042118	.0024258
Σ_{a_2}	.0076572	.0061825	.0046432	.015135	.0072539	.0042326
$\Sigma_{r_{1\to2}}$	9.4752×10^{-3}	9.7259×10^{-3}	1.008×10^{-2}	8.4228×10^{-3}	9.7886×10^{-2}	1.0449×10^{-2}
$\Sigma_{r_{2\to1}}$	8.1492×10^{-6}	6.7641×10^{-6}	5.1952×10^{-6}	1.5144×10^{-5}	7.9316×10^{-6}	4.8935×10^{-6}
ξ	.028488	.033672	.042703	.031303	.049513	.075380
C^{28}/F^{25}	.66499	.66329	.66143	.50762	.50024	.49750

TABLE A-III

TWO GROUP DATA FOR MARK 5R LATTICES (RAHABR)

Lattice Pitch Constant	5.5"	6.0"	7.0"	8.0"
$\bar{\alpha}_1{}^{25}$	.44146	.43966	.43471	.42891
$\bar{\alpha}_2{}^{25}$	.17619	.17510	.17361	.17266
$\bar{\nu}_1{}^{25}$	2.4253	2.4260	2.4276	2.4296
$\bar{\nu}_2{}^{25}$	2.4188	2.4188	2.4188	2.4188
$\bar{\nu}_1{}^{28}$	2.8079	2.8067	2.8046	2.8010
D_1	1.1697	1.1862	1.2100	1.2252
D_2	.86328	.85546	.84246	.83250
Σ_{f_2}	.013153	.010826	.0076081	.0055121
Σ_{a_2}	.023495	.019291	.013523	.0097922
$\Sigma_{r_{1 \to 2}}$	6.2702×10^{-3}	6.8441×10^{-3}	7.7079×10^{-3}	8.3090×10^{-3}
$\Sigma_{r_{2 \to 1}}$	2.2185×10^{-5}	1.8652×10^{-5}	1.3650×10^{-5}	1.0275×10^{-5}
ξ	.040974	.040611	.041356	.043783
C^{28}/F^{25}	.57646	.57214	.56637	.56269

TABLE A-IV

TWO GROUP DATA FOR MARK 15 LATTICES (RAHABR)

Lattice Pitch / Constant	6.35"	7.0"	8.08"	9.25"
$\bar{\alpha}_1^{25}$	.44046	.43766	.43189	.42500
$\bar{\alpha}_2^{25}$	.17625	.17506	.17369	.17277
$\bar{\nu}_1^{25}$	2.4257	2.4265	2.4283	2.4304
$\bar{\nu}_2^{25}$	2.4188	2.4188	2.4188	2.4188
$\bar{\nu}_1^{28}$	2.8084	2.8068	2.8048	2.8032
D_1	1.1887	1.2040	1.2217	1.2341
D_2	.86264	.85336	.84061	.83058
Σ_{f_2}	.014065	.011143	.0077843	.0055145
Σ_{a_2}	.023242	.018373	.012814	.0090783
$\Sigma_{r_{1 \to 2}}$	6.3820×10^{-3}	6.9825×10^{-3}	7.7297×10^{-3}	8.3129×10^{-3}
$\Sigma_{r_{2 \to 1}}$	2.2530×10^{-5}	1.8342×10^{-5}	1.3394×10^{-5}	9.9302×10^{-6}
ξ	.029357	.029766	.031568	.034803
C^{28}/F^{25}	.44994	.44632	.44220	.43937

TABLE A-V

TWO GROUP DATA FOR CRNL ZEEP LATTICE (RAHABR)

Pitch Constant	20 cm
$\bar{\alpha}_1^{25}$	.41332
$\bar{\alpha}_2^{25}$	.17046
$\bar{\nu}_1^{25}$	2.4404
$\bar{\nu}_2^{25}$	2.4188
$\bar{\nu}_1^{28}$	2.8031
D_1	1.2682
D_2	.81113
Σ_{f_2}	.0018192
Σ_{a_2}	.0034463
$\Sigma_{r_{1 \to 2}}$	1.0214×10^{-2}
$\Sigma_{r_{2 \to 1}}$	3.9567×10^{-6}
ξ	.052663
C^{28}/F^{25}	.66982